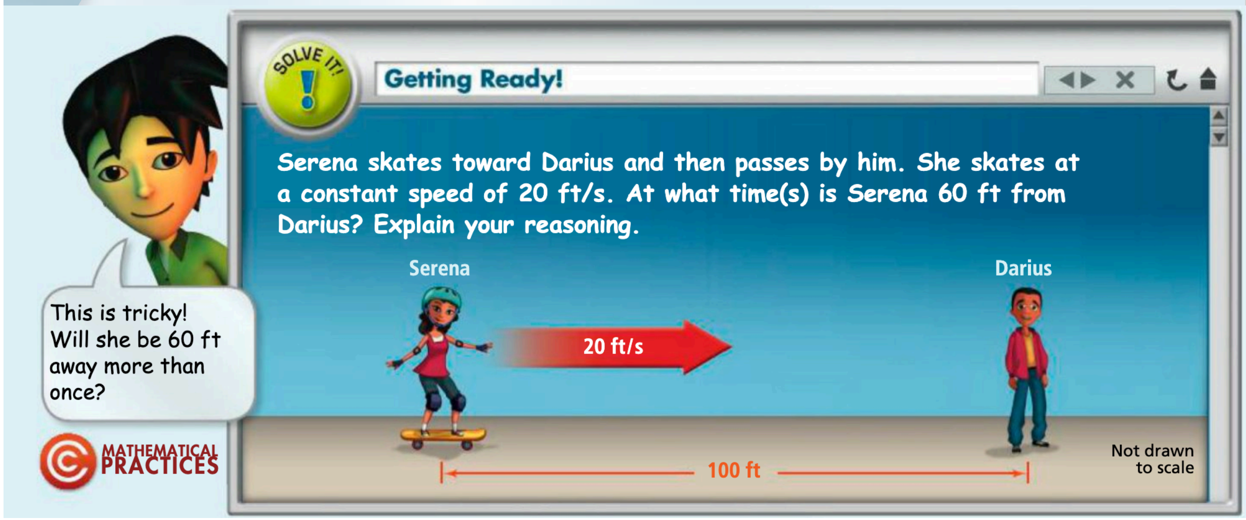
3-7 Absolute Value Equations and Inequalities



In the Solve It, Serena's distance from Darius decreases and then increases. You can use absolute value to model such changes.

You can solve absolute value equations and inequalities by first isolating the absolute value expression, if necessary. Then write an equivalent pair of linear equations or inequalities.

PROBLEM 1: SOLVING AN ABSOLUTE VALUE EQUATION

Solve each equation. Check your solutions.

a) $|x| + 2 = 9$

b) $|n| - 5 = 2$

c) $-3|m| = -9$

d) $|t| - 3 = 7$

e) $5|r| = 20$

f) $\frac{|y|}{7} = 3$

Some equations, such as $|2x - 5| = 13$, have variable expressions within absolute value symbols. The equation $|2x - 5| = 13$ means that the distance on a number line from $2x - 5$ to 0 is 13 units. There are two points that are 13 units from 0: 13 and -13. So to find the values of *x*, solve $2x - 5 = -13$ and $2x - 5 = 13$. You can generalize this process as follows:

<u>KEY CONCEPT: SOLVING ABSOLUTE VALUE EQUATIONS</u>

To solve an equation in the form $|A| = b$, where A represents a variable expression and $b > 0$, solve:

$\boldsymbol{A = -b}$ **and** $\boldsymbol{A = b}$

PROBLEM 2: SOLVING AN ABSOLUTE VALUE EQUATION

Solve each equation. Check your solutions.

a) $|r - 8| = 5$

b) $-3|2w| = -12$

c) $|4f + 1| - 2 = 5$

d) $|m - 2| = 3$

e) $|3t - 2| - 6 = 2$

f) $4|2y - 3| - 1 = 11$

Recall that an absolute value represents distance from 0 on a number line. Distance is always nonnegative. *So any equation that states that the absolute value of an expression is negative has no solution.*

PROBLEM 3: SOLVING AN ABSOLUTE VALUE EQUATION WITH NO SOLUTION

Solve each equation. Check your solutions. If there is no solution, write no solution.

a) $3|2x + 9| + 12 = 10$

b) $-4|k| = 12$

c) $|3x - 6| - 5 = -7$

d) $-6|2x - 3| = -30$

You can write absolute value inequalities as compound inequalities. Think about some numbers that would make each one of these inequalities true.

$|n| < 2$ $\qquad\qquad\qquad\qquad$ $|n| > 3$

KEY CONCEPT: SOLVING ABSOLUTE VALUE INEQUALITIES

To solve an inequality in the form $|A| < b$, where A is a variable expression and $b > 0$, solve the compound inequality $-b < A < b$.

To solve an inequality in the form $|A| > b$, where A is a variable expression and $b > 0$, solve the compound inequality $A < -b \text{ or } A > b$.

Similar rules are true for $\leq$ and $\geq$.

PROBLEM 4: SOLVING AN ABSOLUTE VALUE INEQUALITY INVOLVING $\geq$

Solve each inequality. Graph the solution.

a) $|8\mathrm{n}| \geq 24$

b) $|\mathrm{p} - 7| > 3$

c) $|5m - 9| \geq 24$

d) $|2x + 4| \geq 5$

e) $|y + 8| > 7$

f) $|3d - 7| > 28$

PROBLEM 5: SOLVING AN ABSOLUTE VALUE INEQUALITY INVOLVING $\leq$

Solve each inequality. Graph your solution.

a). $|x - 2| < 5$

b) $|2c - 5| \leq 9$

c) $|2v - 1| \leq 9$

d) $|x + 3| < 1$

e) $|2f + 9| \leq 13$

f) $2|x - 3| + 5 \leq 15$

PROBLEM 6: REAL-WORLD PROBLEM SOLVING

a) Starting at 100 feet away, your friend skates toward you and then passes by you. She skates at a constant speed of 20 ft/s. Her distance *d* from you in feet after *t* seconds is given by $d = |100 - 20t|$. At what times is she 40 feet away from you?

b) A company makes boxes of crackers that should weigh 213 g. A quality-control inspector randomly selcts boxes to weigh. Any box that varies from the weight by more than 5 g is sent back. What is the range of allowable weights for a box of crackers?